# Methodology

The figure below briefly illustrates the methodology for analysing the statistical properties of the Woods Point aftershock sequences and the modelled duration.A diagram of a system

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Figure 1. A schematic summarising the methodology.

## 1.1 Gutenberg-Richter Law

The Gutenberg-Richter Law (GR Law) is a fundamental scaling relationship that describes earthquake characteristics based on a statistical model of rock and crustal deformation (e.g., Scholz, 1968). This law illustrates the distribution of earthquakes across various magnitudes, where the frequency of earthquakes follows a power-law distribution with respect to magnitude (Equation 1).

Equation 1

Where is the cumulative number of earthquakes with a magnitude greater than or equal to . The value represents regional seismicity (i.e., M>=0), while the value indicates the relative proportion of earthquakes across different magnitude bins. The global average value is approximately 1, suggesting that for each increase of one unit in earthquake magnitude, the frequency of earthquakes decreases by a factor of ten.

Previous studies have shown that the b value estimated using the Maximum Likelihood Estimation (MLE) method is closer to the actual b value than that obtained using the Least Squares Method (LSQ) (Felzer, 2006; Estay et al., 2020). The MLE method calculates the b value using the following equation (Aki, 1965):

Equation 2

Where represents the mean magnitude, represents the minimum magnitude cutoff value above which the catalogue is considered well-recorded, and indicates the magnitude bin width (e.g., 0.1 in this study). The MLE method is particularly relevant to magnitude distribution because it calculates the mean magnitude difference for probability, influenced by magnitude bins with the maximum population. In contrast, the LSQ method may overlook the skewness of the error distribution. After estimating the b value, its uncertainty can be calculated using the following equation (Shi & Bolt, 1982):

Equation 3

Where is obtained from Equation 1, and represents the number of events in the catalogue above the .

In addition to the influence of method selection on b value estimation, accurately estimating b values relies on well-recorded earthquake catalogues. An incomplete catalogue can introduce bias into the b value by affecting the Frequency-Magnitude Distribution. To address the issue of magnitude incompleteness—characterised by the underrepresentation of small magnitude events due to limitations in the sensitivity and spatial coverage of seismic instruments—a magnitude completeness threshold (Mc) can be applied. This cutoff magnitude separates well-recorded earthquakes from the original catalogue. However, bootstrapping errors, caused by sampling bias due to the stochastic nature of higher magnitudes in short-duration time windows, may affect the magnitude distribution and, consequently, the determination of Mc. In this study, we employed different approaches to address these uncertainties and selected a relatively complete catalogue.

**Catalogue inaccuracy at a small magnitude**. For Mc estimation in the Woods Point catalogue, two methods are used: the Maximum Curvature (MAXC) method (Wyss et al., 1999; Wiemer & Wyss, 2000) and the Mc estimation based on b-value stability (MBS) method (Cao & Gao, 2002).

a) The MAXC method identifies the point of maximum curvature in the observed Frequency-Magnitude Distribution, which corresponds to the magnitude bin with the highest frequency. This occurs because, as the catalogue reaches the detection level, the number of events in that magnitude bin will peak, indicating accurate recording. However, this method does not account for scenarios where the detection threshold of seismometers coincides with a specific magnitude bin, causing that bin to artificially peak. Additionally, the MAXC method may underestimate the Mc value (Mignan & Woessner, 2012; Wiemer & Wyss, 2000). Despite these drawbacks, the MAXC method is a simple and straightforward approach to obtain Mc and provides relatively stable results even with a limited catalogue size (Mignan et al., 2011).

b) The MBS method operates on the assumption that the b value will remain stable and close to its true value if the catalogue is complete (i.e., M ≥ Mc), as the frequency of earthquakes follows a power-law decrease (i.e., the b value) according to the Gutenberg-Richter (GR) law (Cao & Gao, 2002). Thus, the Mc value is identified as the first magnitude at which the b value stabilises. The stability of the b value is measured using the following formula:

Equation 4

This means the b value is considered stable when the difference between the estimated b value (e.g., using the MLE method) and the average b value is within the b value uncertainty. The is calculated as follows:

Equation 5

In this study, the magnitude bin width is 0.1, and is equal to 0.5. Therefore, the represents the average b value calculated over the continuing half-magnitude range from a given starting magnitude using the MLE method (Equation 2), with the starting magnitude assumed to be the magnitude cutoff.

However, the Mc estimated from b-value stability may be overestimated compared to the actual Mc value (Woessner & Wiemer, 2005; Mignan & Woessner, 2012). Therefore, combining the Mc values estimated from both the MAXC and MBS methods may provide a more accurate range for Mc. Additionally, using a higher magnitude cutoff can help create a well-recorded catalogue, as larger magnitude events release significant energy, making them easier for the permanent network to detect accurately.

Moreover, the completeness of earthquake catalogues is often compromised in the first 48 hours after a mainshock due to the interference of overlapping coda waves (Hainzl, 2016) and the lack of nearby seismometers, which can reduce the detection accuracy of small magnitude events.

**Sampling errors at a large magnitude.** Sampling errors can introduce bias into the Frequency-Magnitude Distribution (FMD), and these errors associated with large-magnitude events can also impact Mc estimation. To address incompleteness issues due to insufficient time span and spatial limitations, the catalogue will be resampled 10,000 times (without any magnitude cutoff) using the bootstrapping method (Efron & Tibshirani, 1994). The MBS method will then be applied to each resampled catalogue. In this process, the Mc value is determined as the average Mc value across all the regenerated datasets, with the standard deviation of the Mc distribution representing the uncertainty in the Mc value.

To estimate the b value for background seismicity, considering the potential misestimation of the occurrence rate of large-magnitude events due to the small study area and limited earthquake productivity, the global average b value (approximately 1) will be used as the baseline for the b-value stability method. However, for the b value of the Woods Point aftershock sequence, the average b value will be calculated using the original method (i.e.,  calculated from the successive magnitude range), with a 0.5 magnitude increment for the calculation.

## 1.2 Omori’s Law (Temporal Distribution)

Omori's Law describes the distribution of aftershock sequences over time. Proposed by Omori (1894), it quantifies how the number of earthquakes decreases with time following a mainshock—the frequency of aftershocks decreases hyperbolically over time. Later, Utsu (1961) modified Omori's Law to account for variations in aftershock decay rates. The modified version of Omori's Law (Equation 6) is now the most commonly used approach in aftershock analysis:

Equation 6

Here, defines the power-law decay rate; if the parameter is to be 1, the cumulative number of aftershock sequences towards infinity as the duration of the aftershock sequence increases (Kagan, 2011). is the productivity of aftershock sequences, which depends on the magnitude of the mainshock (Hainzl & Marsan, 2008). is the time elapsed from immediately after the mainshock until the aftershock sequence begins to follow a power-law decay (M ≥ Mc); the value reflects the duration of incomplete aftershock catalogues due to coda wave overlap and data acquisition failures (Vidale et al., 2004; Peng et al., 2006).

There are two methods to calculate the parameters of Modified Omori’s Law:

**Maximum Likelihood Method**: Proposed by Ogata (1983), this method assumes that aftershock sequences follow a non-stationary Poisson distribution. It estimates parameters by maximizing the likelihood of a non-stationary Poisson model using the aftershock time series.

**Bayesian Analysis**: Due to the interconnections among the three parameters, estimating any one of them inaccurately can affect the overall accuracy of Omori's Law. Holschneider et al. (2012) addressed this issue by redefining the parameters within a Bayesian framework to eliminate internal dependencies.

## 1.3 Spatial Distribution

### 1.3.1 Aftershock spatial distribution

Theoretically, regional seismicity should gradually return to its pre-mainshock state. While the seismicity rate triggered by the mainshock can be quantified using Omori's Law, the spatial distribution of seismic events is expected to initially differ from the pre-mainshock stage and then gradually transition back to the background seismicity uniform pattern. By comparing the distance distribution of events from the mainshock hypocentre during the post-mainshock stage with that of the pre-mainshock stage, we can evaluate the current state of the aftershock sequence. This comparison can specifically help determine whether regional seismicity has reverted to its background state.

### 1.3.2 Aftershock Energy Released Distribution

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Figure 2. A schematic summarising the methodology for calculating the energy release distribution from near-source aftershock sequences (i.e., aftershocks occurring within 10 km of the mainshock rupture plane).

Most aftershocks are believed to trigger the rupture of fractures surrounding the mainshock fault, rather than re-rupturing the mainshock fault itself (Yukutake & Iio, 2017). The spatial distribution of aftershocks may reflect the geometry of the fault plane that ruptured during the mainshock, based on the observed spatial aftershock sequences (Grimm et al., 2022). Consequently, the spatial distribution of aftershock sequences is related to the fault plane involved in the mainshock. After identifying this fault plane (see Appendix 2 for the methodology to obtain rupture planes from near-source aftershock sequences), the spatial distribution of aftershock energy can be analyzed. The method for investigating this energy distribution involves two steps (Figure 2): dataset preparation and analysis of the spatial distribution of released energy.

The dataset preparation includes calculating the moment magnitude (Mo) for each event (e.g., Equation S3) and converting coordinates from the earthquake's geographical coordinates to a metre-based coordinate system. This metre-based coordination is beneficial for the next step, which involves calculating the relative distance from the mainshock rupture plane to each aftershock's hypocentre. Distance calculation can be simplified by rotating the metre-based coordinate system to align with the rupture plane. This rotation method, used during rupture plane fitting from the aftershock sequence, is detailed in Appendix 2.

To analyse the spatial distribution of energy released from near-source seismicity (i.e., aftershocks occurring within a maximum distance of 10 km from the fault plane), we employ a binning method (Figure 3). The bin length starts at 0 m and increases by 1 m in each iteration until it reaches a maximum of 10 km. For each bin, we calculate the energy released by aftershocks within its boundaries. The boundaries of each bin are equidistant from the four edges of the fault plane. This method allows us to plot the energy released by near-source aftershocks relative to their distance from the fault plane.

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Figure 3. This diagram illustrates the binning method used to calculate the cumulative seismic moment of aftershocks within each bin. The bin is represented as a cubic shape, with the yellow plane indicating the fault plane. Each face of the cubic is equidistant from the edges of the fault plane.

## 1.4 Bath’s Law

Bath’s Law, introduced by Båth in 1965, asserts that the difference in magnitude between the mainshock and its largest aftershock typically falls within a fixed range, with a mean difference of 1.2 (Båth, 1965; Apostol, 2021). However, this law was originally based on empirical observations and did not incorporate considerations of magnitude scaling or aftershock productivity (Helmstetter & Sornette, 2003).

Shcherbakov & Turcotte (2004) extended Bath’s Law by integrating the Gutenberg-Richter Law. In this modified version, the magnitude of the largest aftershock is estimated based on the magnitude at which the cumulative number N(≥M) reaches 1, as per the Gutenberg-Richter Law (Equation 1). The modified magnitude difference is given by:

Equation 7

Where is the magnitude of the mainshock. is the largest aftershock inferred from the Gutenberg-Richter Law.

The mean difference for the modified Bath’s Law is 1.11, which is lower than the mean difference of 1.2 observed in the original Bath’s Law. Additionally, the modified version demonstrates a reduced standard deviation of 0.29 in the magnitude difference, compared to 0.46 for the original law. This reduction in standard deviation enhances the accuracy of predictions for the largest aftershock.

## 1.5 Duration of Aftershock sequence

According to Omori's Law, aftershock sequences decay over time. Short-term aftershock durations, characterised by distinct clustering properties in both space and time, can be identified from earthquake records. Two methods are commonly used for estimating short-term aftershock sequence durations based on these clustering properties:

1. **Seismicity Rate Method**. This approach compares the background seismicity rate (pre-mainshock seismicity rate) with the seismicity rate observed since the mainshock to determine if the current rate falls within the background range. The seismicity rate following the mainshock can be estimated using Omori's Law. However, catalogue uncertainties due to sampling errors and detection sensitivity may lead to an incomplete catalogue, introducing bias into the background seismicity calculation. To address this, a synthetic catalogue generated using the Gutenberg-Richter relationship from the region prior to the mainshock is used to calculate the background seismicity rate. This method helps predict when regional seismic activity transitions back to its long-term behaviour, consistent with the pre-mainshock stage.
2. **Cumulative Energy Released Method.** This method addresses the limitations of the Seismicity Rate Method, particularly its reliance on catalogue completeness. By evaluating the monthly seismic energy released, it reduces the impact of poorly recorded small-magnitude earthquakes, especially in the first 48 hours post-mainshock and during periods of poor network coverage. The approach involves calculating the cumulative energy released each month before the mainshock and establishing a high-confidence range for background seismicity using the probability density function. This range is then used to assess the monthly cumulative energy after the mainshock. If the current energy release falls within this background range, the aftershock sequences are considered to have transitioned to the long-term stage.

# References

Aki, K. (1965). Maximum likelihood estimate of b in the formula log10N = a - bm and its confidence limits. Bulletin of Earthquake Research, 43, 237-239.

Apostol, B. F. (2021). Correlations and Bath’s law. Results in Geophysical Sciences, 5, 100011. <https://doi.org/10.1016/j.ringps.2021.100011>

Båth, M. (1965). Lateral inhomogeneities of the upper mantle. Tectonophysics, 2(6), 483-514. <https://doi.org/10.1016/0040-1951(65)90003-X>

Cao, A., & Gao, S. S. (2002). Temporal variation of seismic b-values beneath northeastern Japan island arc. Geophysical Research Letters, 29(9), 48-1-48-3. <https://doi.org/10.1029/2001GL013775>

Efron, B., & Tibshirani, R. J. (1994). An introduction to the bootstrap. Chapman and Hall/CRC. <https://doi.org/10.1201/9780429246593>

Estay, R., Vallejos, J., Pavez, C., & Brönner, M. (2020). A comparison of characteristic parameters of mining-related and tectonic seismic aftershock sequences. International Journal of Rock Mechanics and Mining Sciences, 128, 104242. <https://doi.org/10.1016/j.ijrmms.2020.104242>

Felzer, K. R. (2006, December). Calculating the Gutenberg-Richter b value. In AGU Fall Meeting Abstracts (Vol. 2006, pp. S42C-08). Available at <https://drive.google.com/file/d/1ctA8o2wks1x1sajPbLkcrmNSVSsyD44f/view?usp=sharing>

Hainzl, S. (2016). Rate‐dependent incompleteness of earthquake catalogs. Seismological Research Letters, 87(2A), 337–344. <https://doi.org/10.1785/0220150211>

Hainzl, S., & Marsan, D. (2008). Dependence of the Omori-Utsu law parameters on main shock magnitude: Observations and modeling. Journal of Geophysical Research, 113(B10). <https://doi.org/10.1029/2007jb005492>

Helmstetter, A., & Sornette, D. (2003). Båth's law derived from the Gutenberg-Richter law and from aftershock properties. Geophysical Research Letters, 30(20). <https://doi.org/10.1029/2003GL018186>

Holschneider, M., Narteau, C., Shebalin, P., Peng, Z., & Schorlemmer, D. (2012). Bayesian analysis of the modified Omori law. Journal of Geophysical Research: Solid Earth, 117(B6). <https://doi.org/10.1029/2011jb009054>

Kagan, Y. Y. (2011). Random stress and Omori’s law. Geophysical Journal International, 186(3), 1347–1364. <https://doi.org/10.1111/j.1365-246x.2011.05114.x>

Mignan, A., & Woessner, J. (2012). Estimating the magnitude of completeness for earthquake catalogs. Community Online Resource for Statistical Seismicity Analysis. <https://doi.org/10.5078/corssa-00180805>. Available at [http://www.corssa.org](http://www.corssa.org/)

Mignan, A., Werner, M. J., Wiemer, S., Chen, C.-C., & Wu, Y.-M. (2011). Bayesian estimation of the spatially varying completeness magnitude of earthquake catalogs. Bulletin of the Seismological Society of America, 101(3), 1371-1385. <https://doi.org/10.1785/0120100223>

Ogata, Y. (1983). Estimation of the parameters in the modified Omori formula for aftershock frequencies by the maximum likelihood procedure. Journal of Physics of the Earth, 31(2), 115-124. <https://doi.org/10.4294/jpe1952.31.115>

Omori, F. (1894). On the aftershocks of earthquakes. Journal of the College of Science, Imperial University of Tokyo, 7, 111-120.

Peng, Z., Vidale, J. E., & Houston, H. (2006). Anomalous early aftershock decay rate of the 2004 Mw6.0 Parkfield, California, earthquake. Geophysical Research Letters, 33(17). <https://doi.org/10.1029/2006GL026744>

Scholz, C. H. (1968). The frequency-magnitude relation of microfracturing in rock and its relation to earthquakes. Bulletin of the Seismological Society of America, 58(1), 399–415. <https://doi.org/10.1785/BSSA0580010399>

Shcherbakov, R., & Turcotte, D. L. (2004). A modified form of Bath's law. Bulletin of the Seismological Society of America, 94(5), 1968-1975. <https://doi.org/10.1785/012003162>

Shi, Y., & Bolt, B. A. (1982). The standard error of the magnitude-frequency b value. Bulletin of the Seismological Society of America, 72(5), 1677-1687. <https://doi.org/10.1785/BSSA0720051677>

Utsu, T. (1961). A statistical study on the occurrence of aftershocks. Geophysical Magazine, 30, 521-605.

Vidale, J. E., Peng, Z., & Ishii, M. (2004, December). Anomalous aftershock decay rates in the first hundred seconds revealed from the Hi-net borehole data. In AGU Fall Meeting Abstracts (Vol. 2004, pp. S23C-07).

Wiemer, S., & Wyss, M. (2000). Minimum magnitude of complete reporting in earthquake catalogs: examples from Alaska, the Western United States, and Japan. Bulletin of the Seismological Society of America, 90(4), 859–869. <https://doi.org/10.1785/0119990114>

Woessner, J., & Wiemer, S. (2005). Assessing the quality of earthquake catalogues: Estimating the magnitude of completeness and its uncertainty. Bulletin of the Seismological Society of America, 95(2), 684-698. <https://doi.org/10.1785/0120040007>

Wyss, M., Hasegawa, A., Wiemer, S., & Umino, N. (1999). Quantitative mapping of precursory seismic quiescence before the 1989, M 7.1 off-Sanriku earthquake, Japan. Annals of Geophysics, 42(5). <https://doi.org/10.4401/ag-3765>